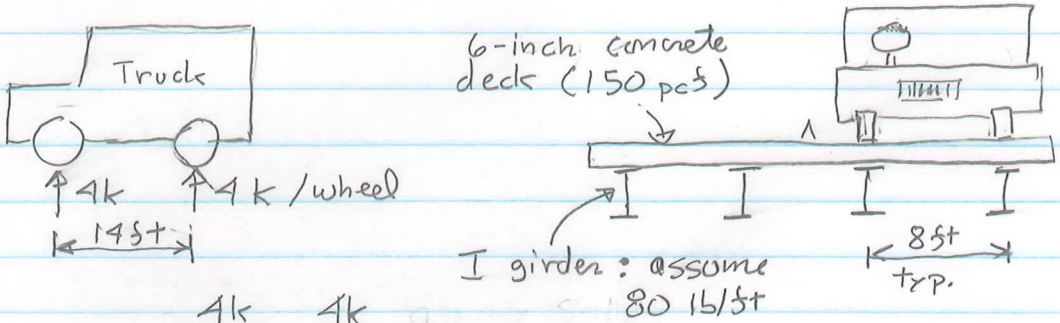
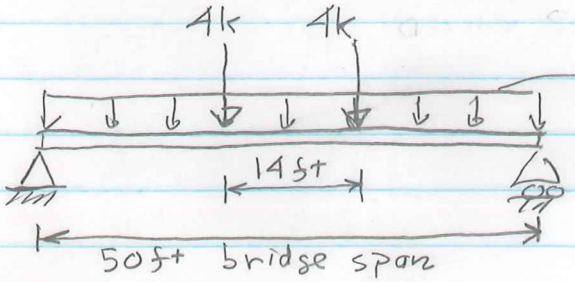
19. Analysis and design of beams for bending

Example: Design the girders of a bridge for a truck loading. For purposes of this example, we will assume only one truck is on the bridge per lane. Assume bending stresses σ_x control the design (not shear stresses).





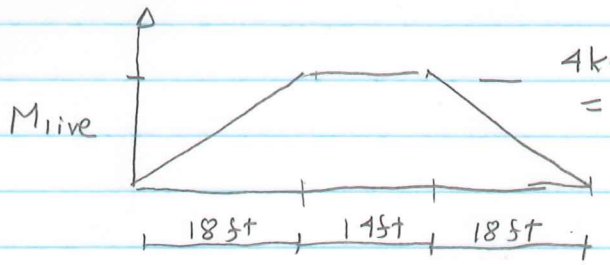
$150\text{pcf} \cdot \frac{1}{2}\text{ft} \cdot 8\text{ft} = 0.600\ \text{k/ft}$

$+\ 0.080\ \text{k/ft}$

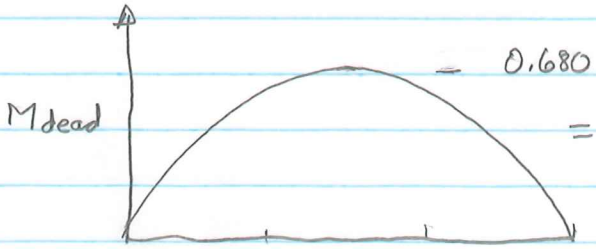
dead load $\rightarrow$ $0.680\ \text{k/ft}$

The truck (live load) is placed in the middle of the bridge to produce the maximum moment.

Draw moment diagrams.



$4\text{k} \cdot 18\text{ft} = 72\ \text{kip-ft}$



$= 0.680\ \text{k/ft} \cdot \frac{(50\text{ft})^2}{8}$

$= 212.5\ \text{kip-ft}$

M_{max} occurs at mid-span $= 284.5\ \text{kip-ft}$

Try allowable stress design, $\sigma_{allow} = 24$ ksi

$$S_{required} = \frac{M_{max}}{\sigma_{allow}} = 142 \text{ in}^3$$

Choose W18×76 with an S of 146 in^3 (page 750 of text)

76 lb/ft, OK

Try ultimate strength design (LRFD)

$\gamma_D = 1.25$, $\gamma_L = 1.75$, $\phi = 0.90$, $\sigma_Y = 36$ ksi

$$M_{dead,max} \cdot \gamma_D + M_{live,max} \cdot \gamma_L \leq \phi M_p$$

$$212.5 \text{ kip-ft} \cdot 1.25 + 72 \text{ kip-ft} \cdot 1.75 \leq 0.9 \cdot M_p$$

$$M_p \geq 435 \text{ kip-ft}$$

$$Z_{required} = \frac{M_p}{\sigma_Y} = 145 \text{ in}^3$$

Choose W16×77 with a Z of 150 in^3

77 lbs/ft, OK

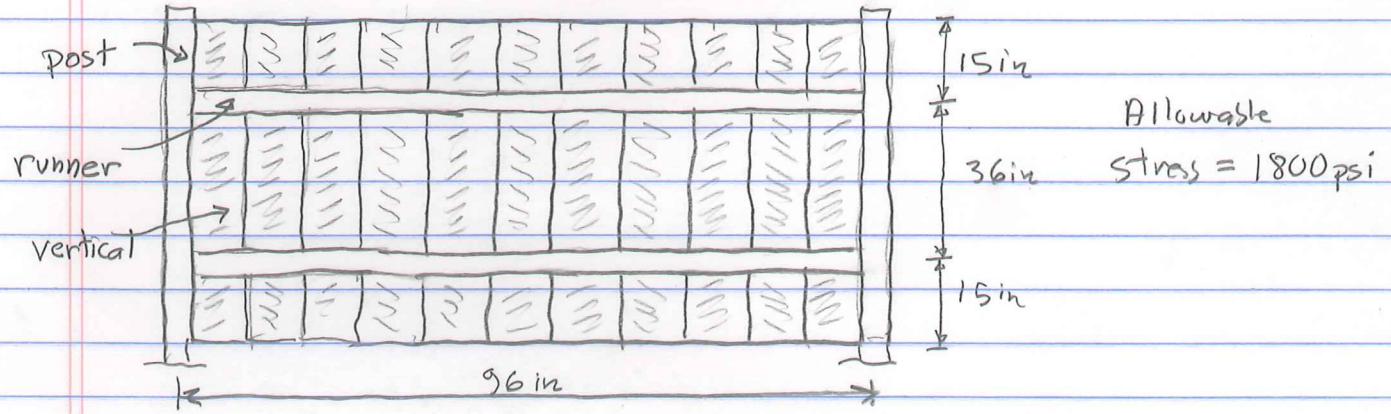
We would also want to check the shear stresses and the bridge deflection.

Fence Design

Use 60 mph wind speed. To convert wind speed to pressure, use

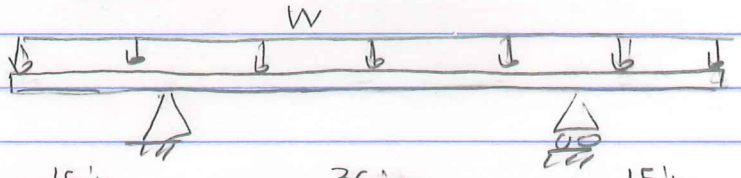
$p = 0.00256\ V^2$ where p is in psf and V is in mph. Get

$p = 9.216$ psf

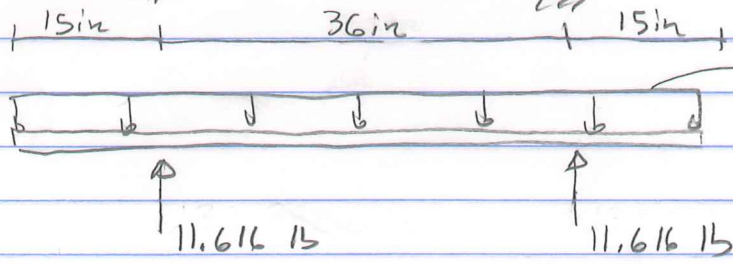


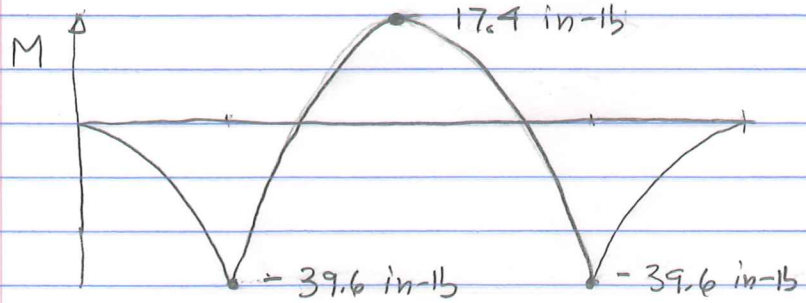
Vertical

Try a 1×6 — $5\frac{1}{2}$ in × $\frac{3}{4}$ in $I = 0.1934\ in^4$



$w = p \cdot \frac{5.5}{12} = 4.224$ lb/ft

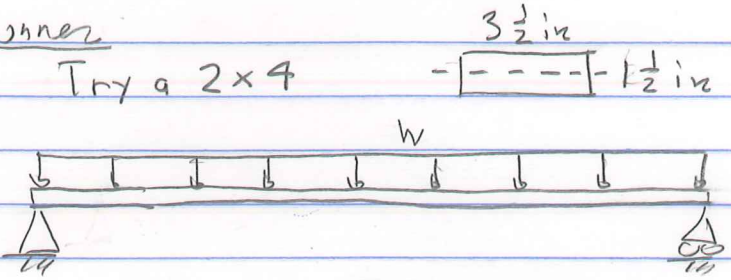




$\sigma_{max} = 39.6\ \text{in-lb} \cdot \frac{3}{8}\ \text{in} / I = 77$ psi (small, OK)

Runner

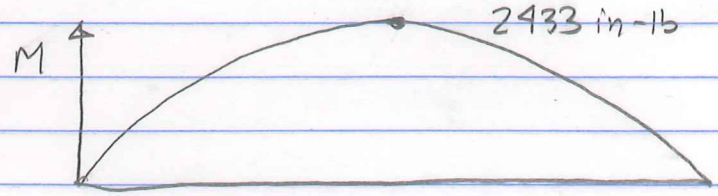
Try a 2×4



$I = 0.984\ in^4$

$w = 11.616\ lb / 5\frac{1}{2}\ in$

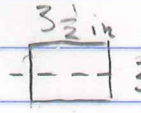
$= 2.112\ lb/in$



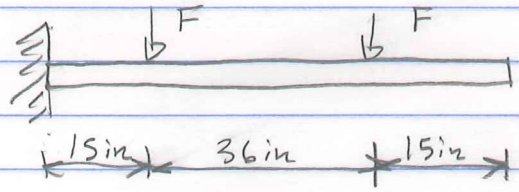
$\sigma_{max} = 2433\ in\text{-}lb \cdot \frac{3}{4}\ in / I = 1854\ psi$ (close; OK)

Post

Try a 4×4

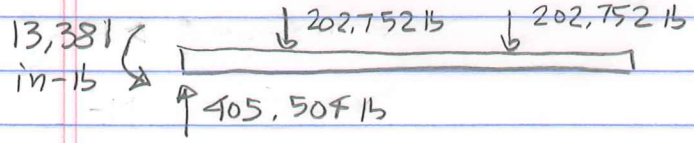


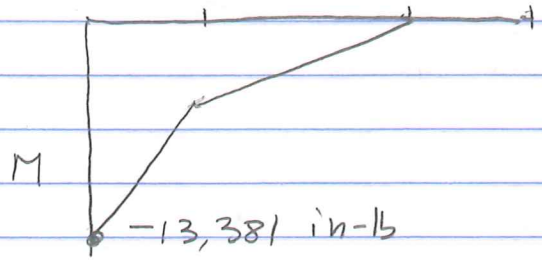
$I = 12.505\ in^4$



$F = 101.376\ lb \times 2$

$= 202.752\ lb$





$\sigma_{max} = 13{,}381\ in\text{-}lb \cdot 1\frac{3}{4}\ in / I = 1873\ psi$ (close; OK)